## Fractions to decimals and vice versa

Six schools choose to report the proportions of their Year 11 pupils obtaining 5 or more GCSE passes at grade C or above in a variety of ways:

School A: three-fifths of the pupils School B: 0.75 of the pupils
School C: 13 pupils in every 25 School D: 0.625 of the pupils
School E: 119 pupils out of 197 School F: 0.59 of the pupils
a) Express the proportions for $\mathrm{A}, \mathrm{C}$ and E in decimal notation. (Only use a calculator if necessary.)
b) Express all the proportions in fraction notation.
c) Which notation is the easier to use to put these proportions in order of size?

## Answers to check-up 8

a) $\mathrm{A}: 0.6$.
C: 0.52.
E: 0.604 approximately.
b) $\mathrm{A}: \frac{3}{5}$
B: $\frac{3}{4}$
C: $\frac{13}{25}$
D: $\frac{5}{8}$
E: $\frac{119}{197}$
F: $\frac{59}{100}$.
c) It is much easier to put the decimal versions in order: $0.52,0.59,0.6,0.604$, $0.625,0.75$.

## Discussion and explanation of check-up 8

To change a fraction to decimal notation, first of all try mentally to change it to an equivalent fraction with a denominator (bottom number) of 10,100 or 1000 - i.e. a power of 10 . You change a fraction to an equivalent fraction by multiplying or dividing both the numerator and the denominator by the same number.

School A: three-fifths $=\frac{3}{5}=\frac{6}{10}$ (multiplying top and bottom by 2 ) $=0.6$ (because $0.6=6$ tenths).

School C: 13 in every $25=\frac{13}{25}=\frac{52}{100}$ (multiplying top and bottom by 4 ) $=0.52$ (because $0.52=52$ hundredths).

The 119 out of 197 for School E cannot be done in this way, because there is no obvious, simple way of getting from 197 to a power of 10 . In this case the fraction is changed to decimal notation simply by dividing the numerator by the denominator, using a calculator: $119 \div 197=0.60406091371$. This can then be rounded to an appropriate number of figures after the decimal point.

To change from decimal notation to fraction notation, express the decimal as so many tenths, hundredths or thousandths, depending on whether there are one, two or three figures after the decimal point. (It is unlikely that there would be any reason to express in fraction notation a decimal number with more than three figures after the point.) Then divide top and bottom by whatever factors they have in common to obtain the simplest equivalent fraction this process is called cancelling. For example, $0.8=\frac{8}{10}=\frac{4}{5}$ (cancelling 2, which is a factor of both 8 and 10).

School B: $\quad 0.75=\frac{75}{100}=\frac{3}{4}$ (cancelling 25, which is a factor of both 75 and 100 ).
School D: $\quad 0.625=\frac{625}{1000}=\frac{125}{200}($ cancelling 5$)=\frac{5}{8}($ cancelling 25$)$.

School F: $\quad 0.59=\frac{59}{100}$ (this cannot be simplified; 59 and 100 have no common factors).

It is usually much easier to compare proportions expressed in decimal notation (or as percentages) than in fraction notation. I cannot compare $\frac{3}{5}$ and $\frac{119}{197}$ at a glance, but I can immediately see that 0.6 is less than 0.604 (or that $60 \%$ is less than 60.4\%).

## Summary of key ideas

- To change a fraction to an equivalent fraction, multiply or divide both the numerator and the denominator by the same number.
- Dividing both numerator and denominator by a common factor is called cancelling.
- To change a fraction to decimal notation, first of all try mentally to change it to an equivalent fraction with a denominator of 10,100 or 1000.
- To change from decimal to fraction notation, express the decimal as tenths, hundredths or thousandths, depending on the number of figures after the decimal point, and then cancel as much as possible.
- It is easier to compare proportions expressed in decimal notation or as percentages than in fraction notation.


## Further practice

8.1 Without recourse to a calculator, express the following in decimal notation, compare the answers with Further Practice question 1.1 (from Check-up 1), and then commit them all to memory!
$\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{20}, \frac{3}{20}, \frac{7}{20}, \frac{9}{20}$
8.2 Change (a) 0.175 and (b) 0.007 to fraction notation.
8.3 One subject department in a secondary school has spent four-sevenths of their equipment budget; another has spent $£ 879$ out of $£ 1500$. Using a calculator, decide which has spent the larger proportion of their budget.

